PVC PIPE news

THE EFFECTS OF DEFLECTION ON SEWER HYDRAULICS
The Sequel

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The sequel tag to the title of this article stems from the fact that *The Effects of Deflection on Sewer Hydraulics* was featured in the Summer 1989 issue of this publication. Those of you who enjoy analytical geometry and integrals should download the earlier version from the newspaper section of our website. If your skills in those areas are a bit rusty, the sequel may be more to your liking.

The topic then, as it is now, is how to tackle the elliptical cross section of a flexible pipe when performing hydraulic calculations for gravity sewers. Every text on hydraulics discusses circular pipe, but those that also cover pipes with an elliptical cross section are in the extreme minority.

Circular Cross Sections

First, a quick review of pipes with a circular cross section. The Manning's equation is the one most often used for calculating the flow velocity. Equation 1 is the imperial version of that formula. After determining the velocity, the flow rate is calculated using Equation 2. In this form, the flow is in units of ft^3/s.

Equation 1

$$v = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

Equation 2

$$Q = vA$$

Where:
- v = velocity of flow, ft/s
- n = Manning's roughness coefficient
- r = hydraulic radius, ft
- s = slope
- A = flow cross sectional area, ft^2

Calculating the hydraulic radius of a circular pipe flowing full is a simple matter. Take the ratio of the cross sectional area of the flow to the wetted perimeter of the pipe in contact with the flow. Let *d* be the inside diameter of the circular pipe and let *p* be the perimeter. Also, use the subscript *c* to signify that these geometric properties are for a circle.

Equation 3

$$A_c = \frac{\pi}{4} d^2$$

Equation 4

$$p_c = \pi d$$

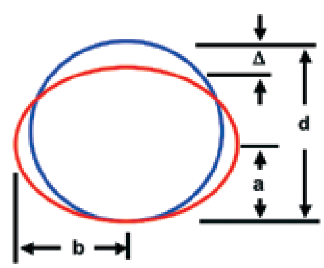
Equation 5

$$r_c = \frac{A_c}{p_c} = \frac{\frac{\pi}{4}(d^2)}{\pi d} = \frac{d}{4}$$

Elliptical Cross Sections

Figure 1 shows the circular cross section of the pipe in blue and the elliptical cross section in red. Let *a* denote the altitude of the ellipse and *b* denote the base.

Figure 1



With this terminology, the cross sectional area and perimeter for an ellipse may be defined. The subscript *e* will be used to identify elliptical geometric properties. The formula for the perimeter for the ellipse is the one listed in the sixth edition of the Civil Engineering Reference Manual, which is a good approximation. The 1989 article goes into more details on calculating the perimeter of an ellipse (using integrals and analytical geometry).

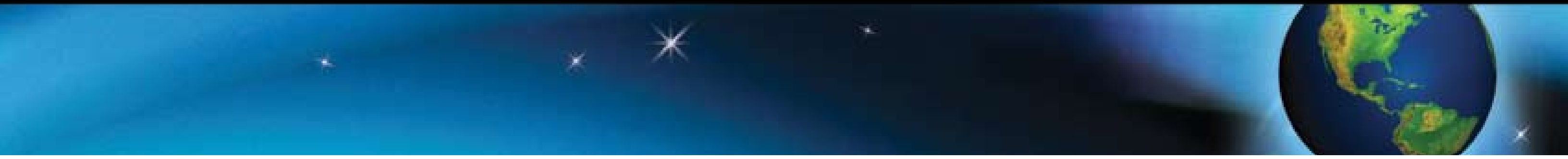
Equation 6

$$A_e = \pi ab$$

Equation 7

$$p_e = 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$$

As the pipe deforms from a circle into an ellipse, the perimeter stays the same for reasonable levels of deflection. Thus, p_c is equal to p_e.



5% Deflection

Let's use a deflection of 5% to demonstrate the concept. With that ring deflection, *d* is decreased by 5%, *2a* is equal to *0.95d*, which means *a* equals *0.475d*. Next, increase the value of *b* until the perimeter of the ellipse matches that of the circle. That occurs when *b* is *0.524d*. Both *a* and *b* may now be written in terms of *d* in Equation 6. This is done below. A_C is also shown for reference.

$$A_E = \pi(0.475D)(0.524D) = 0.249\pi D^2$$

$$A_c = 0.250\pi D^2$$

Taking the ratio of the two areas, one finds that the cross sectional area of this ellipse is only slightly less than that of a circle - just 0.4%. Getting the perimeter and hydraulic radius of this ellipse in terms of *d* takes a bit more effort.

$$p_e = 2\pi\sqrt{\frac{1}{2}\left\{(0.475d)^2 + (0.524d)^2\right\}} = 1.002\pi d$$

$$r_e = \frac{A_e}{p_e} = \frac{0.249\pi d^2}{1.002\pi d} = 0.249d$$

The ratio of flow in the deflected pipe to the circular pipe is shown below. From that ratio, one determines that the reduction in flow is also minimal - just 0.7%.

$$\frac{A_e(r_e)^{\frac{2}{3}}}{A_c(r_c)^{\frac{2}{3}}} = \frac{(0.249\pi d^2)(0.249d)^{\frac{2}{3}}}{(0.250\pi d^2)(0.250d)^{\frac{2}{3}}} = 0.993$$

SUMMARY

Table 1 was developed using the more rigorous approach for the perimeter as described in the 1989 article. As the table shows, in the allowable deflection range (7.5% or less), the effects of deflection on sewer hydraulics are not a concern.

Table 1

EFFECTS OF DEFLECTION ON SEWER HYDRAULICS

Deflection (%)	Reduction in A_e Compared to A_c (%)	Reduction in Q_e Compared to Q_c (%)
5	0.37	0.6
7.5	0.81	1.4
10	1.43	2.4
15	3.15	5.2
20	5.47	9.0
25	8.38	13.6
30	11.8	18.9
35	15.8	24.9